FACULTY OF ENGINEEERING

BE III Semester (CBCS((Backlog) Examination, November 2021

Subject: Engineering Mathematics - III

Time: 2 Hours

Max. Marks: 70

(Missing data, if any, may be suitably assumed)

PART – A

Note: Answer any five questions. **(5x2=10 Marks)**

1. Show that $f(z) = xy + iy$ is everywhere continuous but is not analytic.
2. State Cauchy's integral formula for derivatives.
3. Show that $f(z) = \frac{\sin z}{z}$ has removable singularity.
4. Expand $f(z) = \frac{1}{z}$ about $z = 2$ in Taylor's series.
5. Express $f(x) = x$ as half- range sine series in $0 < x < 2$.
6. Write Fourier series expansion of even periodic function $f(x)$ in $(-c, c)$.
7. Form the partial differential equation by eliminating the arbitrary function from $z = (x + y)\emptyset(x^2 - y^2)$.
8. Solve $xp + yq = 3z$.
9. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$.
10. Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ using the separation of variables.

PART – B

Note: Answer any four questions. **(4x15=60 Marks)**

11. a) Find 'k' such that $f(x, y) = x^3 - 3kxy^2$ is harmonic and find its harmonic conjugate.
    b) Evaluate $\oint \frac{e^{2z}}{(z+i)^4} dz, C: |z| = 3$, using Cauchy's integral formula

12. a) Evaluate $\oint \frac{ze^z}{(z^2+9)} dz$, where c is $|z| = 5$.
    b) Find the bilinear transformation which maps the points $(1, i, -1)$ of z – plane to $(2, i, -2)$ of w – plane.

13. Find the Fourier series expansion of following periodic function $f(x)$ of period 4
$$f(x) = \begin{cases} 2 + x, & -2 \le x \le 0 \\ 2 + x, & 0 \le x \le 2 \end{cases}$$ hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$

14. a) Find the general solution of the partial differential equation
$$(y + z)p + (z + x)q = x + y.$$
    b) Solve $(D^2 + 4DD' - 5D'^2)z = \sin(2x + 3y)$.

15. An elastic string of length l which is fastened at its end $x = 0$ and $x = l$ is picked up at its center point $x = l/2$ to a height of h and released from rest. Find the lateral displacement of the string at any instant time.

16. a) Expand $f(z) = \frac{1}{(z^2 - 3z + 2)}$ as Laurent's series in the region
(i) $0 < |z - 1| < 1$ (ii) $0 < |z| < 2$.
b) Solve $2\sqrt{p} \quad 3\sqrt{q} = 6x + 2y$.

17. a) Show that $f(z) = \bar{z}$ is continuous at the point $z = 0$ but not differentiable at $z = 0$.
b) Express $f(x) = \frac{x}{2}$ as a Fourier series in $-\pi < x < \pi$.

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